1　式の計算

Practice 1

(1)

$$
\begin{aligned}
\text{与式} &= a^3 + a^2b - ac^2 - ab^2 + bc^2 - b^3 \\
&= -(a-b)c^2 + a^3 - b^3 + a^2b - ab^2 \\
&= -(a-b)c^2 + (a-b)(a^2+ab+b^2) + ab(a-b) \\
&= (a-b)(-c^2 + a^2 + 2ab + b^2) \\
&= (a-b)\{(a+b)^2 - c^2\} \\
&= (a-b)\{(a+b)+c\}\{(a+b)-c\} \\
&= (a-b)(a+b+c)(a+b-c)
\end{aligned}
$$

(2)

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\begin{aligned}
\text{与式} &= \frac{1 - \dfrac{1}{1-\dfrac{1}{1+x}} \cdot \dfrac{1+x}{1+x}}{1 - \dfrac{1}{1-\dfrac{1}{1-x}} \cdot \dfrac{1-x}{1-x}} \\
&= \frac{1 - \dfrac{1+x}{1+x-1}}{1 - \dfrac{1-x}{1-x-1}} \\
&= \frac{1 - \dfrac{1+x}{x}}{1 - \dfrac{1-x}{-x}} \\
&= \frac{1 - \dfrac{1+x}{x}}{1 - \dfrac{1-x}{-x}} \cdot \frac{x}{x} \\
&= \frac{x-(1+x)}{x+(1-x)} \\
&= \frac{-1}{1} \\
&= -1
\end{aligned}
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\begin{aligned}
与式 &= 6x^2-(y+7)x-2y^2+7y-3 \\
&= 6x^2-(y+7)x-(2y-1)(y-3) \\
&= \{3x-(2y-1)\}\{2x+(y-3)\} \\
&= (3x-2y+1)(2x+y-3)
\end{aligned}
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\begin{aligned}
与式 &= \{-a+(b+c)\}\{a+(b+c)\}\{a-(b-c)\}\{a+(b-c)\} \\
&= \left\{-a^2+(b+c)^2\right\}\left\{a^2-(b-c)^2\right\} \\
&= -a^4+a^2(b-c)^2+(b+c)^2a^2-(b+c)^2(b-c)^2 \\
&= -a^4+a^2\left\{(b-c)^2+(b+c)^2\right\}-\{(b+c)(b-c)\}^2 \\
&= -a^4+a^2\left(2b^2+2c^2\right)-\left(b^2-c^2\right)^2 \\
&= -a^4+2a^2b^2+2a^2c^2-\left(b^4-2b^2c^2+c^4\right) \\
&= -a^4-b^4-c^4+2a^2b^2+2b^2c^2+2c^2a^2
\end{aligned}
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2

(1)

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\begin{aligned}
与式 &= \left\{\left(x^2+y^2\right)+xy\right\}\left\{\left(x^2+y^2\right)-xy\right\}\left(x^4-x^2y^2+y^4\right) \\
&= \left\{\left(x^2+y^2\right)^2-(xy)^2\right\}\left(x^4-x^2y^2+y^4\right) \\
&= \left(x^4+x^2y^2+y^4\right)\left(x^4-x^2y^2+y^4\right) \\
&= \left\{\left(x^4+y^4\right)+x^2y^2\right\}\left\{\left(x^4+y^4\right)-x^2y^2\right\} \\
&= \left(x^4+y^4\right)^2-\left(x^2y^2\right)^2 \\
&= x^8+x^4y^4+y^8
\end{aligned}
$$

(2)

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\begin{aligned}
与式 &= \left\{(a+b+c)^2-(b+c-a)^2\right\}+\left\{(c+a-b)^2-(a+b-c)^2\right\} \\
&= \{(a+b+c)+(b+c-a)\}\{(a+b+c)-(b+c-a)\} \\
&\qquad +\{(c+a-b)+(a+b-c)\}\{(c+a-b)-(a+b-c)\} \\
&= (2b+2c)\cdot 2a+2a(-2b+2c) \\
&= 8ac
\end{aligned}
$$

3

(1)

$$与式=\{(x+1)(x+4)\}\{(x+2)(x+3)\}-24$$
$$=(x^2+5x+4)(x^2+5x+6)-24$$
$$=\{(x^2+5x)+4\}\{(x^2+5x)+6\}-24$$
$$=(x^2+5x)^2+10(x^2+5x)$$
$$=(x^2+5x)(x^2+5x+10)$$
$$=x(x+5)(x^2+5x+10)$$

(2)

$$与式=(4x^4+16)+7x^2$$
$$=\{(2x^2+4)^2-16x^2\}+7x^2$$
$$=(2x^2+4)^2-9x^2$$
$$=\{(2x^2+4)+3x\}\{(2x^2+4-3x)\}$$
$$=(2x^2+3x+4)(2x^2-3x+4)$$

(3)

$$与式=(ab-c)d+a^2bc-ac^2-ab^2+bc$$
$$=(ab-c)d+ac(ab-c)-b(ab-c)$$
$$=(ab-c)(d+ac-b)$$
$$=(ab-c)(ac-b+d)$$

補足

a についての 2 次式，b についての 2 次式，c についての 2 次式，d についての 1 次式であるから，次数の最も小さい d について整理すると楽。

4

(1)

$$与式=\frac{5-i}{1-3i}\cdot\frac{1+3i}{1+3i}+\frac{9+5i}{3+i}\cdot\frac{3-i}{3-i}$$
$$=\frac{(5-i)(1+3i)}{(1-3i)(1+3i)}+\frac{(9+5i)(3-i)}{(3+i)(3-i)}$$
$$=\frac{5+15i-i-3i^2}{1^2-9i^2}+\frac{27-9i+15i-5i^2}{9-i^2}$$
$$=\frac{5+14i+3}{1+9}+\frac{27+6i+5}{9+1}$$
$$=\frac{8+14i}{10}+\frac{32+6i}{10}$$
$$=\frac{40+20i}{10}$$
$$=4+2i$$

(2)

$$
\begin{aligned}
与式 &= \sqrt{\frac{4+2\sqrt{3}}{2}} + \sqrt{\frac{4-2\sqrt{3}}{2}} \\
&= \sqrt{\frac{\left(\sqrt{3}+1\right)^2}{2}} + \sqrt{\frac{\left(\sqrt{3}-1\right)^2}{2}} \\
&= \frac{\sqrt{\left(\sqrt{3}+1\right)^2}}{\sqrt{2}} + \frac{\sqrt{\left(\sqrt{3}-1\right)^2}}{\sqrt{2}} \\
&= \frac{\left|\sqrt{3}+1\right|}{\sqrt{2}} + \frac{\left|\sqrt{3}-1\right|}{\sqrt{2}} \\
&= \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{3}-1}{\sqrt{2}} \\
&= \frac{2\sqrt{3}}{\sqrt{2}} \\
&= \sqrt{6}
\end{aligned}
$$

5

(1)

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\begin{aligned}
与式 &= \{(x+1)(y+1)\}(xy+1) + xy \\
&= (xy+x+y+1)(xy+1) + xy \\
&= \{(xy+1)+x+y\}(xy+1) + xy \\
&= (xy+1)^2 + (x+y)(xy+1) + xy \\
&= \{(xy+1)+x\}\{(xy+1)+y\} \\
&= (xy+x+1)(xy+y+1)
\end{aligned}
$$

(2)

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\begin{aligned}
与式 &= \left\{(2x)^3 + (3y)^3\right\} + 18xy - 1 \\
&= \left\{(2x+3y)^3 - 3\cdot 2x\cdot 3y(2x+3y)\right\} + 18xy - 1 \\
&= (2x+3y)^3 - 1^3 - 18xy(2x+3y) + 18xy \\
&= \{(2x+3y)-1\}\left\{(2x+3y)^2 + (2x+3y)\cdot 1 + 1^2\right\} - 18xy\{(2x+3y)-1\} \\
&= (2x+3y-1)\left\{(2x+3y)^2 + 2x+3y+1-18xy\right\} \\
&= (2x+3y+1)\left(4x^2-6xy+9y^2+2x+3y+1\right)
\end{aligned}
$$

補足

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\begin{aligned}
(a+b)^3 &= a^3 + b^3 + 3a^2b + 3ab^2 \\
&= a^3 + b^3 + 3ab(a+b)
\end{aligned}
$$

より，

$$
a^3 + b^3 = (a+b)^3 - 3ab(a+b)
$$

(3)

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\begin{aligned}
与式 &= a^4 - 2(b^2 + c^2)\cdot a^2 + b^4 - 2b^2c^2 + c^4 \\
&= (a^2)^2 - 2(b^2 + c^2)\cdot a^2 + (b^2 + c^2)^2 - 4b^2c^2 \\
&= \left\{(a^2)^2 - 2(b^2 + c^2)\cdot a^2 + (b^2 + c^2)^2\right\} - (2bc)^2 \\
&= \left\{a^2 - (b^2 + c^2)\right\}^2 - (2bc)^2 \\
&= (a^2 - b^2 - c^2)^2 - (2bc)^2 \\
&= \left\{(a^2 - b^2 - c^2) + 2bc\right\}\left\{(a^2 - b^2 - c^2) - 2bc\right\} \\
&= \left\{a^2 - (b^2 - 2bc + c^2)\right\}\left\{a^2 - (b^2 + 2bc + c^2)\right\} \\
&= \left\{a^2 - (b-c)^2\right\}\left\{a^2 - (b+c)^2\right\} \\
&= \{a + (b-c)\}\{a - (b-c)\}\{a + (b+c)\}\{a - (b+c)\} \\
&= (a+b-c)(a-b+c)(a+b+c)(a-b-c)
\end{aligned}
$$

6

(1)

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\begin{aligned}
与式 &= \frac{2}{(1+\sqrt{2})+\sqrt{3}}\cdot\frac{(1+\sqrt{2})-\sqrt{3}}{(1+\sqrt{2})-\sqrt{3}} + \frac{2}{(1-\sqrt{2})+\sqrt{3}}\cdot\frac{(1-\sqrt{2})-\sqrt{3}}{(1-\sqrt{2})-\sqrt{3}} \\
&\qquad + \frac{3}{(1+\sqrt{2})-\sqrt{3}}\cdot\frac{(1+\sqrt{2})+\sqrt{3}}{(1+\sqrt{2})+\sqrt{3}} + \frac{3}{(1-\sqrt{2})-\sqrt{3}}\cdot\frac{(1-\sqrt{2})+\sqrt{3}}{(1-\sqrt{2})+\sqrt{3}} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2} + \frac{2(1-\sqrt{2}-\sqrt{3})}{(1-\sqrt{2})^2-(\sqrt{3})^2} + \frac{3(1+\sqrt{2}+\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2} + \frac{3(1-\sqrt{2}+\sqrt{3})}{(1-\sqrt{2})^2-(\sqrt{3})^2} \\
&= \frac{2(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}} + \frac{2(1-\sqrt{2}-\sqrt{3})}{-2\sqrt{2}} + \frac{3(1+\sqrt{2}+\sqrt{3})}{2\sqrt{2}} + \frac{3(1-\sqrt{2}+\sqrt{3})}{-2\sqrt{2}} \\
&= \frac{(1+\sqrt{2}-\sqrt{3})-(1-\sqrt{2}-\sqrt{3})}{\sqrt{2}} + \frac{3\left\{(1+\sqrt{2}+\sqrt{3})-(1-\sqrt{2}+\sqrt{3})\right\}}{2\sqrt{2}} \\
&= 2+3 \\
&= 5
\end{aligned}
$$

(2)

$$
\begin{aligned}
与式 &= \sqrt{9(x^2+4x+4)} - \sqrt{4(x^2-2x+1)} \\
&= 3\sqrt{(x+2)^2} - 2\sqrt{(x-1)^2} \\
&= 3|x+2| - 2|x-1|
\end{aligned}
$$

ここで，$\sqrt{(x+2)^2} \geq 0$，$\sqrt{(x-1)^2} \geq 0$ だから，

$\sqrt{(x+2)^2} = |x+2|$，$\sqrt{(x-1)^2} = |x-1|$

よって，与式 $= 3|x+2| - 2|x-1|$

これより，

$x<-2$ の場合

$$\begin{aligned}3|x+2|-2|x-1| &= 3\{-(x+2)\}-2\{-(x-1)\}\\ &= -3(x+2)+2(x-1)\\ &= -x-8\end{aligned}$$

$-2\le x<1$ の場合

$$\begin{aligned}3|x+2|-2|x-1| &= 3(x+2)-2\{-(x-1)\}\\ &= 3(x+2)+2(x-1)\\ &= 5x+4\end{aligned}$$

$1\le x$ の場合

$$\begin{aligned}3|x+2|-2|x-1| &= 3(x+2)-2(x-1)\\ &= 3(x+2)-2(x-1)\\ &= x+8\end{aligned}$$

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$-x-8$

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$5x+4$